New Fast Precise Kinematic Surveying Method Using a Single Dual-Frequency GPS Receiver

Zhizhao Liu¹; Shengyue Ji²; Wu Chen³; and Xiaoli Ding⁴

Abstract: As of this writing, there are two popular methods to perform precise surveying using the single dual-frequency Global Positioning System (GPS) receiver. One is the Real-Time Kinematic (RTK) technique. The other is the Precise Point Positioning (PPP) technique. The RTK technique requires GPS surveyors to have at least one GPS base station while the PPP technique needs a long observation period to resolve the carrier-phase ambiguities. This paper proposes a new, fast method that can conduct precise kinematic surveying using a single dual-frequency GPS receiver, overcoming the aforementioned problems. This new method, Absolute Plus Loop-based Accumulated-solution Time-relative (APLAT), combines the GPS absolute-positioning method and a loop-based accumulated-solution time-relative positioning method. In this APLAT method, the kinematic surveying trajectory must form a loop. The coordinates of the start-point of the loop can be precisely determined using the absolute positioning function of APLAT by making two short sessions of static observations. This function is useful if no control-point is available in the kinematic surveying. The relative positions of other kinematic surveying points in the loop are determined using the function of loop-based accumulated-solution time-relative GPS positioning. The integration of the absolute and the relative positioning functions allows kinematic surveying to precisely determine the absolute coordinates of each surveyed point in the loop. To determine the absolute coordinates of the start-point of a loop, only two short sessions (5 min each in this study) of static GPS observations (prior to starting and after completing the loop surveying) are required. The results of extensive absolute-positioning tests show that 3D-positioning standard deviation about 7 cm and root-mean-square (RMS) error about 13.4 cm can be achieved when the loop-duration is in the range of 40–80 min. Too short or long a loop-duration may degrade the absolute-positioning accuracy. Results from various relative positioning tests using the loop-based accumulated-solution time-relative method indicate that relative-positioning RMS errors of 1.3 cm, 2.6 cm, and 8.8 cm can be obtained for loop durations of 20 min, 40 min, and 60 min, respectively. This new APLAT method offers a new, fast, and precise (RMS error of 12 cm for absolute and RMS error of 8.8-cm relative positioning for 60-min loop duration) kinematic surveying method that is able to meet many GPS surveying and mapping requirements. This is particularly useful for postmission data processing in circumstances where single-base-station RTK or network RTK service is not available. In this study, the final precise-orbit and precise-satellite-clock data from the International GNSS Service are used to mitigate these errors. At present, this method is useful just for postmission applications, but it offers the flexibility of using absolute-positioning or relative-positioning function only, depending upon the specific surveying and mapping conditions and requirements. This method is a good complement to the PPP and RTK precise-positioning techniques currently in use. Due to the increasing spatial decorrelation of error sources, the positioning accuracy may degrade as the maximum spatial distance (approximately between the loop start-point and the loop middle-point) increases. DOI: 10.1061/(ASCE)SU.1943-5428.0000092. © 2013 American Society of Civil Engineers.

CE Database subject headings: Global positioning systems; Surveys; Kinematics.

Author keywords: Global Positioning System; GPS; Absolute Plus Loop-based Accumulated-solution Time-relative; APLAT; Time-relative; Absolute positioning; Relative positioning.

Introduction

Global Positioning System (GPS) kinematic positioning and navigation with decimeter accuracy is important for a variety of surveying and mapping applications that include land and high-precision marine or hydrographic kinematic applications (Seeber 1993), heave component and shipborne tidal monitoring (Lachapelle et al. 1992; Bonnefond et al. 2003), sea-floor monitoring (Seeber 1993), cadastral surveying, harbor and port mapping (Burrough and McDonnell 1998), coastal resources management and coastal mapping, surveys of sensitive habitats (Johnson and Barton 2004), and satellite altimetry calibration (Crétaux et al. 2008; Bonnefond et al. 2010). Traditionally, the space-relative positioning method is used to achieve centimeter-level accuracy by making simultaneous GPS observations using two GPS receivers at two separated stations, where one is the base station and the other is the rover station. The separation distance between the two stations normally is no more than 30 km for a single baseline (Gao and Wojciechowski 2004; Dodd 2007) and around 70–90 km for network Real-Time Kinematic (RTK) applications (Feng and Rizos 2009). At present, this space-relative positioning method is, to our knowledge, virtually the only widely used approach for high-precision GPS applications.
However, this method needs at least one base station or a GPS network as infrastructure. Meanwhile, data communication between the rover and its base station (or network) is required. Another factor is that the cost of a dual-frequency-with-RTK capability is much higher than one without RTK function. Over the past decade, the Precise Point Positioning (PPP) method has been developed (Zumberge et al. 1997a; Zumberge et al. 1997b). Using the precise orbit and precise satellite-clock corrections, the kinematic PPP can also achieve centimeter-level positioning accuracy; however, it takes several tens of minutes for the carrier phase ambiguities to converge before centimeter-level accuracy is achieved in PPP kinematic surveying (Chen et al. 2004; Gao et al. 2005; Dixon 2006; Geng et al. 2010). Such a long convergence time in PPP constrains its application in many situations where rapid GPS surveying is required and surveying efficiency is concerned.

Over the past decade, a time-relative positioning method, shown in Fig. 1, has been proposed (Ulmer et al. 1995; Michaud and Santerre 2001; Balard et al. 2006; Odijk et al. 2007; Traugott et al. 2010). Distinct from the space-relative positioning method, this method performs differencing in the time domain and it requires only one GPS receiver. To begin, the GPS receiver is placed at station A to collect GPS data for a short session (5 min in this study). After that, it is moved to station B and records GPS observations for another short session (e.g., 5 min). Thus, the relative position between stations A and B can be calculated by performing time-differencing between the two sessions of GPS data. If the coordinates of station A are precisely known, the absolute position of station B can also be determined.

The two kinds of relative positioning methods are somewhat analogous to each other. Both methods can determine the relative position between two stations. The main difference is that for the space-relative method, the differencing is performed between the GPS data collected at the same time by two receivers at two stations and that in the time-relative one, the differencing is performed between the GPS data collected at different times by only one single receiver at two stations. Compared with the traditionally used space-relative method, the advantages of the time-relative one are: only one GPS receiver is required, which significantly reduces GPS-surveying cost and eliminates the labor of deploying a base station; and the carrier-phase ambiguities are cancelled and ambiguity resolution is not required (Balard et al. 2006), which considerably simplifies GPS data-processing. Its disadvantage is that the positioning error may be much greater than the error in the space-relative method. This is because observations at two stations are not obtained simultaneously. Many time-variable errors in GPS signals such as satellite-clock error, orbit error, and atmospheric error have a larger impact on the time-relative solution than on the space-relative solution that is derived from simultaneous observations (Balard et al. 2006). This is probably one of the major reasons preventing the wide application of the time-relative method in practical GPS surveying.

To reduce the effect of time-variable errors on the time-relative positioning solutions, an improved method based on loop misclosure was proposed (Balard et al. 2006). In this improved method shown in Fig. 2, the GPS receiver is required to return to its original station A after surveying station B. Thus, a close-loop is constructed. The close-loop can be split into two halves: the first forward half-loop (station A to station B) and the second backward half-loop (station B to station A). One relative positioning solution can be calculated for each half-loop. The sum of the two solutions in theory should be 0 m, but in practice, it is a nonzero value. It is called the “close-loop misclosure”. This misclosure is largely the result of time-variable errors. By distributing this misclosure, a correction is applied to the relative positioning solution, thus the accuracy of station B can be improved (Balard et al. 2006).

Nevertheless, the accuracy improvement from this loop-based, time-relative positioning method is limited. The positioning accuracy degrades rapidly when the observation duration between stations A and B gets longer (Balard et al. 2006). It was shown that the 3D positioning error could reach about 16 cm within a just 8-min loop with dual-frequency GPS static data (Balard et al. 2006). This improved algorithm is still not practical for many applications that require centimeter or subdecimeter accuracies.

Most recently, an accumulated-solution, time-relative GPS positioning method was proposed (Traugott et al. 2010). In Traugott et al. (2010), the kinematic non-loop accumulated-solution time-relative method was developed. The method was tested under two scenarios: one of an aircraft and the other of a bird, an albatross. A low-cost GPS receiver was employed in this experiment (Traugott et al. 2010).

In contrast to previous studies, this paper develops a new GPS surveying method called Absolute Plus Loop-based Accumulated-solution Time-relative (APLAT). It combines the absolute GPS-positioning method and a loop-based, accumulated-solution, time-relative GPS-positioning method. The capability to estimate the absolute position of station A (the start-point of time-relative positioning) is important because in some GPS applications there are no control points with precise coordinates. The combination of precise absolute positioning and precise time-relative positioning enables the surveyors to obtain precise absolute coordinates of each surveyed point in the loop, even if no control point is available.

This paper is organized as follows: Current Method for Time-Relative Positioning mathematically presents the current method for time-relative positioning and various implementation strategies. The new APLAT method, including absolute positioning function and relative positioning function, is then developed in next section. Numerical Results describes the results of extensive tests and evaluations of the performance of the APLAT method, including both absolute positioning and relative positioning performances. Discussion and Conclusion sections follow.

![Fig. 1. Principle of time-relative positioning method](image1)

![Fig. 2. Loop-based time-relative positioning with overall-solution strategy](image2)
Current Method for Time-Relative Positioning

Mathematical Models for the Time-Relative Positioning Method

In the time-relative positioning method, it is assumed that GPS observation is recorded at station A at epoch $t_0$. At epoch $t_1$, the instrument is moved to station B and GPS observation is recorded again, as shown in Fig. 1. If satellite $S$ is observed at both epochs $t_0$ and $t_1$, the following observation equations can be formed:

\[ A^S_S X_b + B^S_b + M^Z^S Z + T_b = L^S_b \]  
\[ A^S_i X_i + B^S_i + M^Z^S Z + T_i = L^S_i \]  

where $X_b$ and $X_i$ are the coordinates of stations A and B, respectively; $A^S_S$ and $A^S_i$ are their corresponding design matrices at epochs $t_0$ and $t_1$, respectively; $B^S_b$ and $B^S_i$ are ambiguity terms for ionosphere-free carrier-phase combination observation at the two epochs; $M^Z^S$ should be identical if the GPS L1 and L2 carrier phase observations have no cycle slip over the time; $Z$ is tropospheric delay in the zenith direction (Xu 2003); $M^S_S$ and $M^S_i$ are tropospheric mapping functions at epochs $t_0$ and $t_1$, respectively (Niell 1996; Niell 2001; Boehm and Schuh 2004; Boehm et al. 2006); $T_b$ and $T_i$ are the receiver clock-errors at epochs $t_0$ and $t_1$, respectively; and $L^S_b$ and $L^S_i$ are ionosphere-free linear combination carrier-phase observations at epochs $t_0$ and $t_1$, respectively (Hofmann-Wellenhof et al. 2001). Other errors such as satellite orbit-error or satellite clock-error are not listed in the preceding equations, as they will be corrected with precise satellite ephemerides and clock data from the International GNSS Service (IGS) or other models (Melchor 1983; Argus and Helffen 1995; Scherneck and Webb 1998; Banville et al. 2008).

If the interval between $t_0$ and $t_1$ is short, $A^S_S$ and $A^S_i$ as well as $M^S_S$ and $M^S_i$ will be very similar to each other. Their differences are negligible, and as it is valid to approximately assume $A^S_S = A^S_i$ and $M^S_S = M^S_i$, Eq. (3) can be yielded as

\[ A^S_i (X_i - X_b) + (T_i - T_b) = L^S_i - L^S_b \]  

If the position of station A is precisely known, Eq. (3) can be further reduced to

\[ A^S_i X_i + (T_i - T_b) = L^S_i - L^S_b + A^S_i X_b \]  

In the right side of Eq. (4), there are four unknown parameters: three coordinate parameters and one GPS receiver-clock parameter (relative error $T_i - T_b$). When more than four satellites are observed, the parameters can be estimated using a least-squares estimator.

Two Implementation Strategies: Overall-Solution and Accumulated-Solution

In the time-relative GPS positioning method, there are two implementation strategies as shown in Fig. 3. One is called the overall-solution, as shown in Fig. 3(a), and the other is the accumulated-solution, as shown in Fig. 3(b) (Traugott et al. 2010).

Overall-Solution

Eq. (3) or Eq. (4) mathematically illustrates a typical overall-solution of the time-relative positioning method. In the overall-solution strategy shown in Fig. 3(a), the relative position at epoch $t_1$ with respect to epoch $t_0$ is estimated using Eq. (3) or Eq. (4) based on the observations recorded at these two epochs only: $t_0$ and $t_1$. Observations recorded at any between-epoch, $t_k (t_k < t_b < t_i)$, are not used in this strategy.

In the overall-solution, the number of common-view satellites between the two epochs may decrease if the time span ($t_1 - t_0$) is relatively large (e.g., 30 min). Our data analysis shows that, normally, 1–2 satellites drop after 30 min of observation. The accuracy of the relative positioning solution will be affected due to the degraded geometry resulting from fewer common-view satellites (Xu 2003).

Accumulated-Solution

On the other hand, in the accumulated-solution, the relative position at a given epoch $t_1$ with respect to the start epoch $t_0$ is an accumulating result of many small incremental relative positions, as shown in Fig. 3(b). At each between-epoch ($t_k < t_i < t_l$), a small incremental relative position between the epoch $t_k$ and its immediate previous epoch $t_k-1$ is estimated. The sum of all the small increments will produce the relative position of epoch $t_i$ with respect to epoch $t_0$. Mathematically, the accumulated-solution, relative-positioning method can be described as follows.

Assuming that a total of $n$ epochs of observations are recorded for satellite $S$, the observation equations can be written as in Eq. (5). The notations in Eq. (5) are the same as those found in Eq. (1):

\[ A^S_i X_1 + B^S + M^Z^S Z + T_1 = L^S_1 \]  
\[ A^S_i X_2 + B^S + M^Z^S Z + T_2 = L^S_2 \]  
\[ \ldots \ldots \]  
\[ A^S_i X_{i-1} + B^S + M^Z^S Z + T_{i-1} = L^S_{i-1} \]  
\[ A^S_i X_i + B^S + M^Z^S Z + T_i = L^S_i \]  
\[ \ldots \ldots \]  
\[ A^S_i X_n + B^S + M^Z^S Z + T_n = L^S_n \]  

Similar to Eq. (3), one can derive Eq. (6) after performing the time-difference between any two adjacent epochs (Traugott et al. 2010):


\[ A_1^2X_{12} + T_{12} = L_{12}^D \]

\[ A_2^2X_{23} + T_{23} = L_{23}^D \]

\[ \ldots \ldots \]

\[ A_i^2X_{i-1,i} + T_{i-1,i} = L_{i-1,i}^S \]

\[ \ldots \ldots \]

\[ A_n^2X_{n-1,n} + T_{n-1,n} = L_{n-1,n}^S \]

where \( X_{i-1,i} = X_i - X_{i-1} \), \( T_{i-1,i} = T_i - T_{i-1} \), and \( L_{i-1,i}^S = L_i^S - L_{i-1}^S \) \((i = 2, 3 \ldots n)\). In each subequation of Eq. (6), there are four unknowns (i.e., three relative position unknowns and one receiver-clock parameter). In Eq. (6), there are a total of \( 4(n-1) \) unknowns. If the number of tracked satellites is \( p \) at each epoch, the total number of observations will be \( p(n-1) \). Least-squares estimation can be carried out to estimate the unknowns if \( p > 4 \) at each epoch. This condition \((p > 4)\) can normally be satisfied with today's GPS satellite constellation. An elevation-dependent weighting scheme is used in this study. After the incremental relative positions \( X_{i-1,i} \) \((i = 2, 3 \ldots n)\) are estimated, the accumulated-solution relative position between stations A and B is immediately obtained as \( \sum_{i=2}^{n} X_{i-1,i} \).

In the accumulated solution, the time-relative solution is based on two adjacent epochs and the time span is normally small (same as the GPS data interval, e.g., 1 s in kinematic GPS surveying). The unreduced number of common-view satellites helps to maintain the GPS geometry. Thus, the GPS positioning accuracy in the accumulated solution is not as degraded as that in the overall solution.

**Comparison of Overall-Solution and Accumulated-Solution**

In theory, the relative position solutions obtained from the two strategies should be identical, since the geometrical distance between station A and station B is invariably regardless of the positioning strategies. However, the solutions estimated from the two strategies are normally different.

These two strategies assume that the satellites do not move between epochs—note that this is the condition required for the design matrices to be identical for the two epochs. In the overall solution, after any reasonable loop length, this assumption might become invalid and this explains why the position error grows with time in the overall solution. Thus, the overall method is usually used for short-time applications. However, in the accumulated solution, the design matrix is computed for each intermediate epoch, and the two epochs are only 1 s apart (for high-rate observation), so it is reasonable to assume the design matrices are identical between two consecutive epochs.

**Loop-Based Overall-Solution Time-Relative Positioning**

One underlying issue associated with the time-relative GPS positioning method is that its positioning error grows with both time and space, while in the conventional space-relative GPS positioning method, the positioning error normally grows with space only. This is because of the nature of GPS errors. In GPS positioning, some errors such as satellite clock-error and receiver clock-error vary with time. Some other errors such as tropospheric error and ionospheric error vary with time and space. Some types of error, such as orbit-error, vary with time only but have space-impact (different impacts on baselines of different distances). In the conventional space-relative GPS positioning method, the GPS observations are simultaneously recorded at two spaced stations. Through the double-difference algorithm, all the time-variable errors (such as the clock errors) are completely cancelled (Xu 2003). The other errors containing time-variable components (such as ionospheric error, tropospheric error, and orbit error) are largely cancelled, depending on the separation of the two GPS stations. In the time-relative GPS positioning method, however, the observations are made at two different epochs at two spaced stations (except in the static case, where the start- and end-stations are not spaced). Therefore, the errors cannot be cancelled as completely as with the conventional space-relative positioning method, because errors with a time-variable nature are not able to cancel out. Detailed analysis showed that the GPS errors such as orbit error, satellite clock-error, and ionospheric error grow linearly with time (Balard et al. 2006).

To correct the errors that grow with time, a loop concept was proposed (Balard et al. 2006). The loop misclosure was calculated and distributed using a temporal linear-interpolation algorithm (Balard et al. 2006). As shown in Fig. 2, the GPS receiver makes measurements at start-station A. It moves to station B, and then returns to station A. Thus, a loop is formed. Using Eq. (3), the forward relative position from stations A to B \( (X_{AB}) \) and the backward relative position from stations B to A \( (X_{BA}) \) can be calculated. In theory, we have the loop misclosure \( X_{AB} + X_{BA} = 0 \). In practice, however, this misclosure is not zero, due to the existence of time-variable errors. Since it has been shown that the errors grow linearly in time (Balard et al. 2006), the relative position \( \Delta X_{AB} \) \( (X_{AB} + X_{BA}) \) can be improved by distributing the loop misclosure in proportional to the time of surveying \( X_{AB} \) \( (X_{BA}) \) with respect to the entire loop, as given in Eq. (7)

\[ \Delta X_{AB} = X_{AB} + \Delta X_{AB} = X_{AB} + X_{BA} \left( \frac{t_1}{t_1 + t_2} \right) \]  

where the value \( \Delta X_{AB} \) denotes the corrected relative position, the value \( \Delta X_{AB} \) is the correction to \( X_{AB} \), and \( t_1 \) and \( t_2 \) are the surveying times used for the forward half-loop and the backward half-loop, respectively, as shown in Fig. 2.

The method proposed in Balard et al. (2006) is essentially a loop-based, time-relative method implemented with overall-solution strategy. This method is an improvement over the previous non-loop one. However, its relative positioning accuracy is still not high enough, and may significantly degrade with the loop observation time. For instance, when a loop reaches 30 min, the relative-positioning error can reach meter level even after the loop-misclosure correction (Balard et al. 2006). For an 8-min loop with misclosure correction, the dual-frequency GPS positioning error is about 11 cm for the horizontal and 29 cm for the vertical component, with a 95% confidence level (Balard et al. 2006). Therefore, new approaches with better performance should be developed to achieve higher GPS positioning accuracy.

**The New Method: APLAT**

**Relative Positioning of APLAT: Absolute Loop-based Accumulated-solution Time-relative**

A new method called as Absolute Plus Loop-based Accumulated-solution Time-relative (APLAT) is developed in this paper. This method integrates the absolute GPS positioning function and a loop-based, accumulated-solution, time-relative GPS positioning function. This is an improvement over the previous work in two aspects: (1) Previous studies did not provide a method to precisely estimate the absolute position at start-station A. In the work by Balard et al. (2006), a known control-point is chosen as start-station A. In the work by Traugott et al. (2010), the position of start-station A is
estimated through code-based, single-point positioning. Thus, the accuracy of start-station A is about a few meters, which is insufficient for precise GPS surveying. This paper develops a method to estimate the absolute position of station A with the accuracy of a few centimeters. With an accurate start-station A, the absolute positions of all the kinematic surveying points (like station B) can be obtained through the time-relative positioning method. (2) Previous work used the loop-based overall-solution time-relative method (Balard et al. 2006). As discussed in Current Method for Time-Relative Positioning, the overall-solution positioning error grows rapidly with time. As will be shown in this paper, the accumulated-solution has superior performance to the overall solution. In the work by Traugott et al. (2010), both the accumulated-solution and overall-solution methods were discussed and analyzed, but no work on loop-misclosure correction was done to improve the GPS solutions. Our APLAT method is based on the loop-misclosure correction and accumulated-solution. It is expected to produce better GPS surveying accuracies.

The capability of the APLAT method to precisely estimate the absolute position of station A (the start-point) is important. In some GPS applications, there is no control-point with precise coordinates. Thus, the absolute coordinates can be estimated using the APLAT method. The combination of precise absolute positioning and precise time-relative positioning offers GPS surveyors a great deal of convenience for determining the absolute position of each surveying point. In some extended period of time at one station significantly degrades the surveying efficiency. A new way to estimate an absolute position using the APLAT method is proposed in this section based on the loop observations.

In the loop constructed in Fig. 4, supposing that j epochs of GPS static observations are recorded at station A before the receiver moves to the next station B. After the GPS receiver returns to station A from station D, k epochs of static observations are collected again at station A. Taking the satellite S as an example, the observation equations can be written as

\[
\begin{align*}
A_1^S X_1 + B_1^S + M_1^S Z + T_1 & = L_1^S \\
A_2^S X_1 + B_2^S + M_2^S Z + T_2 & = L_2^S \\
& \vdots \\
A_{n-k+1}^S X_1 + B_{n-k+1}^S + M_{n-k+1}^S Z + T_{n-k+1} & = L_{n-k+1}^S \\
A_n^S X_1 + B_n^S + M_n^S Z + T_n & = L_n^S
\end{align*}
\]  

(10)

In Eq. (10), the unknown vector \(X_1\) represents the coordinates of the start station A. The other symbols in Eq. (10) are similar to those in Eq. (5) or (6). Considering that the normal duration of the loop surveying is less than 2 h, the zenith-tropospheric delay can be approximately estimated by using one parameter. \(B^S\) is the ionosphere-free, carrier-phase-bias part related to ambiguity that is considered as constant if any cycle slips have been corrected (Liu 2011). Assume that a total of p satellites are observed throughout the loop period. Thus, the total number of unknowns is \((3 + p + 1 + j + k) = (4 + p + j + k)\). The total number of observations is \(p^S (j + k)\). When \(p^S (j + k) > (4 + p + j + k)\), a least-squares estimation can be performed. With the current GPS satellite constellation, a receiver can normally track \(p = 8\) satellites at a given epoch. If \((j + k) \approx 2\), a sufficient number of observations will be available for conducting least-squares estimation. The total number of epochs \((j + k)\) is usually quite large (e.g., 600 in this study; \(j = 300\) and \(k = 300\), with the data interval as 1 s). This ensures that a large number of redundant observations exist for least-squares estimation to have an accurate and reliable estimation of the coordinate vector \(X_1\).

The principle behind Eq. (10) is the same as that used in static PPP. The only difference is that in this APLAT method, only a small portion of static data is used for coordinate determination. In this study, the number of static epochs at station A is \((j + k) = 600\) epochs, which accounts for only 16.7% of the total epochs \((n = 3600)\). The rest data are kinematic data and they are not collected at station A. The normal duration of the loop is more than 1 h. With such a long duration, the satellite geometry has changed significantly. This changed geometry helps improve GPS surveying accuracy and reliability; it is the rationale behind the idea that this APLAT method can estimate the station-A coordinates precisely with only two short sessions of static observations.

**Absolute Positioning of APLAT: Integration of Loop-Start and Loop-End**

When conducting GPS surveying in the field, it is common for surveyors to have difficulty finding a control-point with known coordinates. In the case of only one GPS receiver and no RTK service or capability, the only way to obtain a precise position seems to be use of the PPP technique. At this stage, the GPS receiver must collect several tens of minutes of data at one station before the carrier-phase ambiguities can converge. Requirement of such an extended period of time at one station significantly degrades the surveying efficiency. A new way to estimate an absolute position using the APLAT method is proposed in this section based on the loop observations.

In the loop constructed in Fig. 4, supposing that \(j\) epochs of GPS static observations are recorded at station A before the receiver moves to the next station B. After the GPS receiver returns to station A from station D, \(k\) epochs of static observations are collected again at station A. Taking the satellite \(S\) as an example, the observation equations can be written as

\[
A_1^S X_1 + B_1^S + M_1^S Z + T_1 = L_1^S \\
A_2^S X_1 + B_2^S + M_2^S Z + T_2 = L_2^S \\
\vdots \\
A_{n-k+1}^S X_1 + B_{n-k+1}^S + M_{n-k+1}^S Z + T_{n-k+1} = L_{n-k+1}^S \\
A_n^S X_1 + B_n^S + M_n^S Z + T_n = L_n^S
\]  

(10)

In Eq. (10), the unknown vector \(X_1\) represents the coordinates of the start station A. The other symbols in Eq. (10) are similar to those in Eq. (5) or (6). Considering that the normal duration of the loop surveying is less than 2 h, the zenith-tropospheric delay can be approximately estimated by using one parameter. \(B^S\) is the ionosphere-free, carrier-phase-bias part related to ambiguity that is considered as constant if any cycle slips have been corrected (Liu 2011). Assume that a total of \(p\) satellites are observed throughout the loop period. Thus, the total number of unknowns is \((3 + p + 1 + j + k) = (4 + p + j + k)\). The total number of observations is \(p^S (j + k)\). When \(p^S (j + k) > (4 + p + j + k)\), a least-squares estimation can be performed. With the current GPS satellite constellation, a receiver can normally track \(p = 8\) satellites at a given epoch. If \((j + k) \approx 2\), a sufficient number of observations will be available for conducting least-squares estimation. The total number of epochs \((j + k)\) is usually quite large (e.g., 600 in this study; \(j = 300\) and \(k = 300\), with the data interval as 1 s). This ensures that a large number of redundant observations exist for least-squares estimation to have an accurate and reliable estimation of the coordinate vector \(X_1\).

The principle behind Eq. (10) is the same as that used in static PPP. The only difference is that in this APLAT method, only a small portion of static data is used for coordinate determination. In this study, the number of static epochs at station A is \((j + k) = 600\) epochs, which accounts for only 16.7% of the total epochs \((n = 3600)\). The rest data are kinematic data and they are not collected at station A. The normal duration of the loop is more than 1 h. With such a long duration, the satellite geometry has changed significantly. This changed geometry helps improve GPS surveying accuracy and reliability; it is the rationale behind the idea that this APLAT method can estimate the station-A coordinates precisely with only two short sessions of static observations.
is particularly promising for situations where there is no network RTK service or where single base-station RTK is not possible. Notably, the nonloop accumulated-solution time-relative method can reach a 3D positioning accuracy of 6.5 cm (shown in Table 3) even if the observation time length is up to 40 min. If the GPS surveyor is interested in only obtaining the relative position, the surveyor is not required to return to the start-point (or to another known control-point) to form a loop. Such flexibility will significantly enhance the efficiency of the GPS surveying.

Note also that the kinematic data analyzed in this paper are obtained from a walking GPS surveyor holding a GPS receiver. The maximum spatial distance (approximately between the loop start-point and the loop middle-point) is a few kilometers. Thus, the spatial-correlated errors are almost completely cancelled in both the absolute and relative positioning of the APLAT method. When this APLAT method is used for GPS data collected over a large area, positioning accuracies may start to degrade due to the increasing spatial decorrelation of the errors.

In short, the APLAT method proposed in this paper provides an economical and technically feasible approach for absolute and/or relatively precise GPS kinematic surveying from using a single GPS dual-frequency receiver. The extensive tests show that the APLAT method can safely offer an accuracy of ~15 cm (absolute plus relative positioning) with a loop duration of 60 min, satisfying the requirements for various surveying and mapping applications. This method is particularly useful for remote regions where RTK may not be available.

**Table 5. Positioning Errors of Loop 3 (in centimeters)**

<table>
<thead>
<tr>
<th>Loop number</th>
<th>Loop duration</th>
<th>Standard deviation improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>North</td>
</tr>
<tr>
<td>1</td>
<td>20 min</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>40 min</td>
<td>70%</td>
</tr>
<tr>
<td>3</td>
<td>60 min</td>
<td>75%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>48.3%</td>
</tr>
</tbody>
</table>

**Acknowledgments**

The authors are grateful for receiving the support from The Hong Kong Polytechnic University projects 1-ZV6L, A-PJ63 and A-PJ78. The International GNSS Service (IGS) is acknowledged for providing some of the data used in this study. The first author thanks the support by the Programme of Introducing Talents of Discipline to Universities (Wuhan University, GNSS Research Center), China. The editor and reviewers for this paper are thanked for their efforts toward improving this manuscript.

**References**


